Written Examination of IGP(C) master course
(Domestic application)
School of Environment and Society
Department of Transdisciplinary Science and Engineering

Hours 13:00～15:00

NOTICE

1. Answer 2 of the three questions among Q[1] ～ Q[3].
2. On each answer sheet, write as "IGP(C)" in the box "試験科目名" and your examination ID number in the box "受験番号". Do not write your name. You do not need to write anything on unused sheets.
3. Do not use scale, compass, and calculator.
4. All answer sheets and draft sheets will be collected after the examination.
5. For each question, use a separate answer sheet.
6. If each question cannot be filled in one answer sheet, you can use multiple sheets.
7. Write the number of each question along with your answer.

Q[1] (differential and integral)

Q[2] (linear algebra)

Q[3] (probability and statistics)

Question [1]

1. Solve the following equations using real numbers.

(1) $f''(t)-f'(t)-6f(t)=0,\quad f(0)=1,\ f'(0)=2$

(2) $f''(t)-4f'(t)+4f(t)=e^{2t},\quad f(0)=0,\ f'(0)=3$

(3) $f''(t)+9f(t)=6\cos 3t,\quad f(0)=1, f'(0)=3$

2. Consider the curved surface of an ellipsoidal body $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1\quad (a>b>c>0)$ to which the cuboid is inscribed with all vertexes. Answer the following questions.

(1) Obtain the volume of the ellipsoidal body V_e.

(2) Find the volume of the cuboid V_c for the case that a vertex in the first octant is denoted by (x,y,z).

(3) In order to obtain the maximum value of (2), by introducing Lagrange's undetermined multiplier λ, obtain the function $f(x,y,z,\lambda)$ for solving the conditional extreme value problem by the following equations.

$$\frac{\partial f}{\partial x}=\frac{\partial f}{\partial y}=\frac{\partial f}{\partial z}=\frac{\partial f}{\partial \lambda}=0.$$

(4) By solving the equations in (3), obtain the maximum volume of the cuboid V_{cmax}.

(5) Compute the maximum volume ratio of the cuboid to the ellipsoidal body $\dfrac{V_{cmax}}{V_e}$ with 3 significant figures.

Question [2]

The transposition and the determinant of a matrix A are denoted by A^T and $|A|$, respectively. The identity matrix is denoted by I. Answer the following questions. Note that the derivation process has to be explained.

1. Answer the values of the following three determinants:

$$\text{(a)} \begin{vmatrix} 2 & 4 \\ 3 & -1 \end{vmatrix} \qquad \text{(b)} \begin{vmatrix} 2 & 4 & -2 \\ 3 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix} \qquad \text{(c)} \begin{vmatrix} 2 & 4 & -2 & 3 \\ 3 & -1 & 1 & 1 \\ 1 & 2 & 1 & 2 \\ 3 & 1 & -1 & 1 \end{vmatrix}$$

2. Let U be an orthogonal matrix. That is, we have $UU^T = U^TU = I$. Answer all possible values of determinants of U. Furthermore, for each of their values, show an example of U.

3. For a natural number N, let A be a real symmetric matrix of order N. Its eigenvalues are denoted by $\lambda_1, \lambda_2, \ldots, \lambda_N$. Then, give $|A|$.

4. If we exchange column vectors of two different columns in a matrix, the value of its determinant is given by multiplying -1 to the determinant of the original matrix. By using this property, show that the value of determinant of a matrix is 0 if column vectors of two different columns in the matrix are the same as the vectors.

5. Solve the following simultaneous linear equations by using Cramer's rule.

$$\begin{aligned} x + 2y + 3z &= 1 \\ 2x - y + z &= -2 \\ 2x + y + 2z &= 2 \end{aligned}$$

Question [3]

The useful life per kilometer of a paved road as the usable time to require repair is approximately described as a normal distribution with a mean of 2.8 years and COV (Coefficient of Variation) of 40%. Assume that the lives among any different one kilometers are statistically independent. Answer the following questions. You can refer to Table 3-1 if needed.

1. What is the probability that one kilometer of paved road will require repair in one year?
2. What is the probability that there will be no repairs required in the first year of a 3-kilometer stretch of paved road?
3. What is the probability that 2 of the 3-kilometer stretch will need repairs in the first year?
4. If 10% is the probability that one kilometer of paved road will require repair, what is the useful life of the road?
5. New pavement materials are introduced in a 25 kilometer section of the paved road as a test. The result of useful life on the road is a mean of 3.2 years and a standard deviation of 12 years as a normal distribution. Define the null hypothesis and the alternative hypothesis whether new pavement materials can change the useful life, and perform a two-sided hypothesis test at the 5% significance level.

Table 3-1 Part of Standard Normal Probability

$P(x < z) = \int_{-\infty}^{z} f(x)\,dx$ $f(x)$: Probability density function of standard normal distribution.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9872	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986